

From Carriage Wheels to Interest Rates: The Evolution of Word Problems in Algebra Textbooks from 1901 to Today

Jemma Lorenat
Pitzer College

Elodie Arbogast

Ethan Baer

Carla Condori Bazan

Robert Bettinger

See next page for additional authors

Follow this and additional works at: <https://scholarship.claremont.edu/jhm>



Part of the [Arts and Humanities Commons](#), and the [Mathematics Commons](#)

Recommended Citation

Lorenat, J. Arbogast, E. Baer, E. Condori Bazan, C. Bettinger, R. Carpenter, E. Davis, H. Grant, D. Howe, O. Kelley, N. Minier, M. Orozco-Valdivia, N. Peck, A. Saavedra, C. Shiwakoty, S. Sidel, H. Stripp, C. Terrien, J. Wolynski, S. and Yearwood, L. "From Carriage Wheels to Interest Rates: The Evolution of Word Problems in Algebra Textbooks from 1901 to Today," *Journal of Humanistic Mathematics*, Volume 10 Issue 1 (January 2020), pages 145-180. DOI: 10.5642/jhummath.202001.08 . Available at: <https://scholarship.claremont.edu/jhm/vol10/iss1/8>

©2020 by the authors. This work is licensed under a Creative Commons License.

JHM is an open access bi-annual journal sponsored by the Claremont Center for the Mathematical Sciences and published by the Claremont Colleges Library | ISSN 2159-8118 | <http://scholarship.claremont.edu/jhm/>

The editorial staff of JHM works hard to make sure the scholarship disseminated in JHM is accurate and upholds professional ethical guidelines. However the views and opinions expressed in each published manuscript belong exclusively to the individual contributor(s). The publisher and the editors do not endorse or accept responsibility for them. See <https://scholarship.claremont.edu/jhm/policies.html> for more information.

From Carriage Wheels to Interest Rates: The Evolution of Word Problems in Algebra Textbooks from 1901 to Today

Authors

Jemma Lorenat, Elodie Arbogast, Ethan Baer, Carla Condori Bazan, Robert Bettinger, Emily Carpenter, Hiawatha Davis III, Derick Grant, Olivia Howe, Neil Kelley, Maya Minier, Naima Orozco-Valdivia, Alan Peck, Carolina Saavedra, Sumesh Shiwakoty, Hunter Sidel, Carter Stripp, Josephine Terrien, Simone Wolynski, and Leana Yearwood

From Carriage Wheels to Interest Rates: The Evolution of Word Problems in Algebra Textbooks from 1901 to Today

Jemma Lorenat, Elodie Arbogast, Ethan Baer, Carla Condori Bazan,
Robert Bettinger, Emily Carpenter, Hiawatha Davis III, Derick Grant,
Olivia Howe, Neil Kelley, Maya Minier, Naima Orozco-Valdivia, Alan Peck,
Carolina Saavedra, Sumesh Shiwakoty, Hunter Sidel, Carter Stripp,
Josephine Terrien, Simone Wolynski, and Leana Yearwood

Pitzer College, Claremont, California, USA
jlorenat@pitzer.edu

Abstract

In teaching algebra, extra-mathematical word problems can bridge the gap between questions about abstract numbers and questions about everyday life. Thus, more than other aspects of elementary algebra, we would expect word problems to have changed in the recent past. This paper documents the findings of a collective research project that examined the content of such word problems over the past century. Alongside amusing and provocative examples, this paper shows how students can participate in exploratory research with primary sources from the history of mathematics.

Key words: word problems; algebra textbooks; 20th century.

Over the past few decades, the use of the history of mathematics in engaging students with mathematics has been explored and analyzed experimentally [8, 29, 31]. Further, recent collaborative lesson-planning projects have shown how teachers of mathematics and its history can fruitfully use historical primary source material to introduce, motivate, and extend the mathematics curriculum (for instance, see <https://blogs.ursinus.edu/triumphs/> to learn about an ongoing project developing relevant classroom materials). Finally, there are successful examples of undergraduates publishing or co-publishing original research in the history of mathematics, including in this

journal. However, research based around a single historical text or event are not readily scalable to the size of a college mathematics classroom. To what extent is it possible for a large body of students (over 20) to collaborate on original historical research?

This question motivated the following research paper in which students from an undergraduate history of algebra class worked together to chart the evolution of word problems in algebra textbooks over the course of the twentieth century and up to the present-day. Word problems are considered an essential part of learning mathematics as one finds on the Common Core Standards website under High School Algebra: “Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling”.¹ In particular, this research focused on the subset of word problems that employ extra-mathematical descriptions and objects as the setting for a quantitative problem. The idea behind the project was that such problems may serve as a small window onto how mathematics textbooks reflect the societies in which they were produced and the people who produced them.

The purpose of publishing this research framed as a project in the history of mathematics is two-fold. First, even while the particular results in this study are not historically rigorous (for reasons described below), the process demanded students to frame and ask questions of primary sources from the history of mathematics. Secondly, we would love to spark a conversation around other collective research projects that might similarly engage student interest, maximize meaningful participation, while somehow circumventing some of the limitations outlined above.

The history of algebra course in which this research was undertaken was designed with no pre-requisites, largely taken by students aiming to fulfill a quantitative reasoning graduation requirement.

For the research project, students signed up for one of six groups, each covering a period of two decades (1901–1920, 1921–1940, and so on, till 2000–2016). For each period there were between six and eight textbooks from which students would find, record, and analyze a sample of word problems. (For a complete chronological list of all texts consulted see Appendix A.) After the analyses were synthesized into a single document, students were

¹<http://www.corestandards.org/Math/Content/HSA/introduction/>, last accessed on January 27, 2020.

encouraged to participate in peer-editing in order to better streamline the approach between sections and look for general overarching trends or striking contrasts.

One major limitation to this project was in the selection of texts, which would accurately be called a sample of convenience drawn from three sources. The main source was the Claremont Colleges Library. From the QA 150s, fifteen texts on beginning / intermediate algebra were identified. Retrospectively, it may have been better to limit the study entirely to this clearly defined sample, but, in the interest of a larger scope balanced across all six periods, additional texts were added. For older texts, a random selection of digitized textbooks was selected through the <http://worldcat.org> database. However, due to copyright restrictions, for texts after the 1980s, we instead relied on supplementing the Claremont Colleges Library collection with books from the Los Angeles Public Library. This shift is problematic in that the potential readers for the three sources are overlapping, but not identical. That said, the algebraic content of both these sets of texts are generally the same: properties of number systems, arithmetic operations for numbers and variables, finding roots of linear and quadratic equations, graphing polynomial equations, properties of exponential and logarithmic equations, and elementary probability and combinatorics. We did not include texts on modern algebra, which are also very well represented in the Claremont Colleges library, but clearly cover different content material.

A more rigorous study would need to draw a much more random and representative sample of textbooks, possibly with reference to the Library of Congress catalog or some other large-scale database. Alternatively, a smaller project might focus exclusively on a small library's collection and could lead to illuminating micro-historical details (some of these emerged in this study and will be highlighted below). It would be very interesting to compare several locally oriented studies of this nature to determine whether there is the same geographic specificity in mathematics word problems as might be seen in (for instance) history textbooks prior to national standardization.

As a result of this patchwork body of texts, some conclusions drawn from this research project may be merely anecdotal. Yet, the broader themes discussed in this article are not unique to this corpus. Word problems are a component of teaching mathematics that date back at least to the problem texts of ancient Mesopotamia and ancient Egypt. In a casual reading, the

problems posed in the Rhind papyrus, for example, look like some of the problems posed in a contemporary elementary algebra textbook. In what follows, caution should be taken in ascribing results of this study to a broader field of word problems or the time periods under study.

If nothing else, word problems (when you don't need to solve them) can be rather fun to read and there is a certain joy in repurposing these miniature literary efforts which might be taught today if not for the power of the lucrative textbook market. For instance, consider the surprisingly long-lived carriage problem as it appeared in the 1920s and 1960s:

The fore wheel of a carriage makes 28 revolutions more than the rear wheel in going 560 yards, but if the circumference of each wheel be increased by 2 feet, the difference would be only 20 revolutions. Find the circumference of each wheel. [13]

In traveling a distance of 175 m the front wheel of a carriage has made 20 more revolutions than the rear wheel, whose circumference is 1 m larger than the circumference of the front wheel. Find the circumference of each wheel. [24]

Further, this research project served to raise several interesting questions and testable hypotheses about the content of such word problems over time. These will be considered in the conclusion.

So much for the historical value; what about the pedagogical impact of such a project? In anonymous course evaluations, students included appreciative comments about being able to collaborate on a research project working towards an outcome that would have been unfeasible at the individual level with the time allotted. However, as is not uncommon in group projects, the work was not always distributed equitably among all group members. Further, once the first draft of the project was completed, it proved difficult to balance necessary edits and additional course material. As a result, the final draft still required substantial revisions, which were helpfully pointed out by anonymous reviewers, but by that time the course was over. Perhaps this might have been avoided by timing the research earlier in the semester, followed by a more rigorous editing process. Certainly, it would have been useful for students to see the gap between a paper that receives an A grade and a paper that gets published.

This timing may not be possible; after all, external reviewers are not bound by the schedule of a 15-week semester. At the very least the project would have been improved with more systematic and detailed documentation of process and evidence. Such a database in progress would have also served as a check against selection bias in emphasizing certain problems that fit preconceptions — for example that problems from the 1940s would contain more references to warfare.

Nevertheless, the research provided an opportunity for students to reflect on their own experience with word problems and whether these appeared as practical, alienating, amusing, or beside the point.

Introductory Overview

In the following we present a mix of problems in the history of American algebra textbooks. Overall, the corpus is conservative. Perhaps unsurprisingly, money remains a dominant theme and problems about distance, rate and time vary in detail, but not in substance. On the other hand, textbook word problems subtly show gradual changes in technology, statistics, and gender dynamics. In order to trace these changes over time, we will proceed chronologically in intervals of two decades as noted (a complete chronological list of all texts consulted can be found in Appendix A). First, it will be helpful to situate a few general patterns regarding audience, style, and content that transcend the temporal divisions.

The books we have used in this study are aimed at three audiences: advanced high school and / or beginning college students, teachers of these students, and “you”—the independent reader who is interested in learning algebra.² Many of the books appear to be intended for two or three of these audiences, and some authors make no mention of audience whatsoever.

²The subject matter was essentially the same for advanced high school or beginning college students. This overlap is noted in W. L. Hart’s introduction to *College Algebra*, where he describes his college audience as students falling into one of the following categories:

1. Students who did not take advanced algebra in high school.
2. Students who took advanced high school algebra but understood the subject so poorly that they should have a complete treatment of intermediate algebra as a part of their freshman course in algebra. [17]

While some authors do specifically emphasize one or more of these audiences in the title or introduction, our research revealed no notable patterns with respect to intended audience and word problem content or style. In fact, many problems persist in practically identical form regardless of whether the author is trying to write a “fun” or a “serious” mathematics book. On the other hand, all three of these audiences changed in composition over the 120 years under consideration here, notably becoming more diverse with respect to race and sex. These changes will be discussed at greater length below.

Our study also suggests three approaches to word problems that may or may not be representative of the larger literature. First, there are authors who avoid extra-mathematical word problems, occasionally with explicit antagonism. Secondly, there are texts that recycle word problems, sometimes with superficial variations of numbers or kinds of objects. Many of these common word problems likely pre-date the period under consideration here, and will likely persist into the future. Though the use of extra-mathematical language suggests a connection to the real world, the scenarios described are not obviously practical and occasionally even absurd to imagine in real life. By contrast, in the third approach, authors take seriously the task of including and analyzing real data with algebra or try to make word problems that align with everyday mathematical encounters. Due to the piecemeal nature of the texts selected, it does not make sense to draw any statistical observations about the distribution of texts into these three categories. Further, there are authors who seem to use a more recycled approach in certain parts of their books, while drawing on real data to explain other concepts. It is notable that no one of these three approaches seem to characterize a particular time period.

For all of the books in this study, the majority of word problems are problems about numbers, equations, graphs, and other mathematical objects. As will be seen, some books have very few or no other kinds of word problems. We have found that the most frequent application of extra-mathematical word problems occur in the context of solving systems of linear equations, comparing percents or fractions, assessing probability, measuring growth or decay over time, and basic combinatorics.

1901–1920

Though textbooks were sampled from the two decades between 1901 and 1920, all of the textbooks in our sample were published in the United States from the years 1901 to 1913. In the same vein, while our corpus was generally made up of algebra textbooks for high school students preparing for college admissions or beginning college students, the textbook written by Stimson Brown was specifically made for midshipmen at the Naval Academy [6]. Yet, overall, the textbooks are similar in terms of intended audience and general content.

Most of the word problems are generic and involve few external details to situate the stated numbers. For instance, there is no investor of the sum in Brown's *Practical Algebra*:

What sum will amount to \$1000 in ten years, interest beginning at $3\frac{1}{2}$ per cent per annum, compounded semi-annually? [6]

It is interesting to note, as we proceed, that the interest earned on investments remains consistently high, and does not seem to reflect banking realities of any particular era. Interest rate problems, as well as problems about buying and selling items, are the most common setting for extra-mathematical word problems in this, as well as all other, time periods. Indeed, since money is a concrete quantity that makes sense as a negative or fractional value, it is particularly well-suited to exhibiting arithmetical operations.

When the problems do become more specific, they most often reference unnamed men performing various actions. The problem form looks familiar, even if the particular items sold do not, such as in this representative example from [41].

A man bought a certain number of railway-shares for \$9375; he sold all but 15 of them for \$10,450, gaining \$20 per share on their cost price; how many shares did he buy? [41]

The buying and selling of railway-shares relates to another problem theme involving transport via boats, trains, carriages, or men moving at different rates and speeds. The 1904 fifth edition of *Algebra: An Elementary Textbook*

by George Chrystal, first published in 1886, employs several problems with walking men.

Two travelers start together on the same road. One of them travels uniformly 10 miles a day. The other travels 8 miles the first day, and increases his pace by half a mile a day each succeeding day. After how many days will the latter overtake the former? [9]

While problems about rate of travel or two objects meeting or passing persist to this day, some specific details from problems of this period reflect an older time, and not only with respect to means and pace of travel. For instance, in a problem from *First Principles of Algebra: Complete Course*, “rods” are used as a distance measurement.

A rectangular field is 32 rods longer than it is wide. The length of the fence around it is 308 rods. Find the dimensions of the field. [40]

This is a historical linear measurement unit, which equals 5.5 yards or approximately 5.029 m.

There is limited use of historical data in problems from this sample. One such example from *Wentworth & Hill's exercise manuals, 2. Algebra* involves a balloon ascension that took place in 1875.

A stone which falls freely passes over respectively in the first three seconds of its fall 4.9^m , 3×4.9^m , and 5×4.9^m . In the famous balloon ascension which was made in 1875, the balloon rose to 8643.6^m . What space would a stone have passed over in the last second falling from the balloon to the ground? [45]

While G. A. Wentworth and G. A. Hill's text was first published in 1884, problems about balloons remained current through H. E. Slaught and N. J. Lennes's *First Principles of Algebra: Complete Course*, first published in 1912.

A balloon which exerts an upward pull of 460 pounds is attached to a car weighing 175 pounds. what is the net upward or downward pull? [40]

The weight of the car is surprising and suggests a different kind of vehicle than what the modern reader might envision.

The persons in problems are always “men” with few exceptions involving generic women.³ The rare woman is usually buying or selling groceries, such as in this problem from [45]

A woman carries to market cabbages for which she receives \$4.20. If she had had 12 cabbages more, and had sold each one at 2 cents more, she would have brought back from market \$2.28 more than she really received. How many cabbages did she have, and what was the selling price per dozen? [45]

Overall, the themes and wording of problems are very similar to today’s word problems. The differences mainly lie in particular details in terms of common objects used in the problems.

1921–1940

With the first fifth of the twentieth-century as a reference, it is easier to identify new technologies and practices within our selection of textbooks between 1920 and 1940. Financial questions remain a significant portion of the problems, but now the investors have names and, if their investments are savvy, might take a flight.

The texts from this period span 1922 to 1937 and were all written for use in the classroom. On occasion this setting was reflected in the word problems themselves, such as in *The Teaching of Elementary Algebra* by Paul Ligda from 1925.

23. A professor of mathematics visiting a class asked the number of pupils enrolled. One of the boys said, “Our number, and the number again, and its half, and its fourth, and one more, makes 100.” [28]

³The presence of women in extra-mathematical word problems is ancient. For instance, there are women in the word problems of the *Suan shu shu* dating back to circa 186 BCE in China.

This question, from a set of exercises on translating “word-statements into symbolic statements” is adjacent to a question about the height of a telegraph pole in a swamp that marks this book, for the modern reader, as from another era. Editions of older texts may have even appeared dated to the student of the 1920s. For instance, the edition of George Chrystal’s *Algebra: Part I* is an identical reprint of the fifth edition first published in 1904 (discussed in the above section as [9]), which in turn is only a slight modification of the 1886 first edition. While a problem about a servant, a master, and payment in farthings, may have appeared realistic to a student in Edinburgh in 1886, the same problem forty years later appears old-fashioned.

A servant agrees to serve his master for twelve months, his wages to be one farthing for the first month, a penny for the second, fourpence for the third, and so on. What did he receive for the year’s service? [10]

While the currency changed, the importance of financial knowledge can be seen in all of these texts. There are many word problems about accounting with significant reference to bank and banking related fields. One typical example from [12] illustrates the general problem style as well as the gender roles of the 1920s’ workplace.

A secretary to a certain business man had to check up on the following interest figures: 5% on \$500 for one year; 6% on \$500 for one year; and $4\frac{1}{2}\%$ on \$500 for one year. How could she compute the interest and not compute each item separately? What was the result? (65)

Problems include calculating interests while making loans, teaching people how to save money, and how to spend money wisely.

Airplanes played a significant role in World War I with commercial aircraft beginning soon afterward. This new transportation technology captured the imagination of textbook writers—five of the seven books consulted contain questions about airplanes. Hart’s *College Algebra* is particularly dedicated to the subject with questions about groundspeed, airspeed, and wireless communication. Notably, the airplanes in Hart’s 1926 text are significantly faster than in the others.

An airplane leaves the deck of a battleship and travels south at the rate of 230 miles per hour. The battleship travels south at the rate of 20 miles per hour. If the wireless set on the airplane has a range of 500 miles, when will the airplane pass out of wireless communication with the ship?

How many fuel hours must be available to permit an airplane flight out from a field for $5\frac{1}{4}$ hours in a direction such that the groundspeed out is 200 miles and back is 185 miles per hour? [17]

By contrast, in W. B. Ford's *A Brief Course in College Algebra*, published in 1922, "two airplanes pass over Chicago, one flying east at 40 miles an hour, the other south at 30 miles an hour" [13]. Moreover, in 1926 — the same year as Hart's 230 mph plane — F. Engelhardt and L. Haertter's airplane flies "at a speed of 90 miles an hour against a wind of 30 miles an hour" [12]. In 1933 Nelson A. Jackson's *Beginning Algebra* features a plane leaving Boston at only 78 mph [20].

The disparity between these values may simply be a result of fictional speeds ungrounded by research. However, it might also be the case that Hart's planes are military aircrafts, while Ford's and Jackson's are commercial planes, thus operating at a much slower speed.

This period also features problems with named persons, which were not apparent in the selection of earlier texts. A full spectrum of the naming continuum can be found in this sample from G. Bartoo and J. Osbourne's review questions [4].

7. Anne's age is eight years more than twice Sue's age. The sum of their ages is twenty. Find the age of each.
8. In five years, John will be four-thirds as old as he is now. What is his present age?
9. Mr. Simpson invests part of his money at four per cent and part at six per cent. The latter amount exceeds the former by \$500. The total annual income is \$230. How much has he invested at four per cent? At six per cent?

There are several additional pairs of names in these questions, but strikingly each problem is limited to a problem about men or women, but not both together.

1941–1960

Given the American involvement in World War II, we initially anticipated that this period might contain more problems pertaining to combat or military technology. However, this was largely not borne out by the examples in these textbooks.

On the one hand, in Frank Millet Morgan's *College Algebra*, produced in 1943, the particularity of the science-related word problems — with mentions of antifreeze, pressure changes in enclosed spaces, bacterium growth in human bodies — demonstrates an attention to new advances, technology, and applications. Yet carriage problems can also be found in this text; a series of exercises on linear functions juxtaposes the traditional with the modern.

21. The rear wheel of a carriage is 3 feet greater in circumference than the front wheel. If the front wheel makes as many revolutions in going a mile as the rear one does in going 2200 yards, find the perimeter of each wheel.
22. How many pints of 80 per cent antifreeze and 40 per cent antifreeze should be mixed to produce 10 quarts of 70 per cent antifreeze? [35]

One wonders whether the persistency of the carriage problem is due to its mathematical interpretation, which cannot be achieved with equi-wheeled modes of transport. Like carriage problems, anti-freeze questions directly connect to questions about mixtures, and re-appear in word problems in later time periods.

Moreover, problems directly addressing conflict or warfare in *College Algebra* are almost timeless in their level of generality. An exercise on quadratic equations involves a besieged garrison and might well apply to any era.

16. A besieged garrison had enough bread to last them 11 days. If there had been 400 more men, each man would have received 2 ounces less per day; if there had been 600 fewer men, each man's daily share could have been increased by 2 ounces, and the garrison would have had enough bread to last them 12 days.

How many pounds of bread did the garrison have, and what was each man's daily share? [35]

Similarly, the description of a marching army invokes a classic warfare motif.⁴

71. As an army 1 mile long began a march, a courier left the rear for the front. He returned, reaching the rear after the army had traveled a mile. How far did the courier travel? [35]

In 1947, Frederick Stanley Nowlan's *College Algebra* also features an army, but here in the context of potential recruitment.

21. Each of 10 men who appear before a medical board with a view to Army enlistment can be dealt with in but two ways, *viz.*, accepted or rejected. (a) In how many ways can exactly 2 men be accepted? (b) In how many ways can exactly 5 men, to include a specified 2 be accepted? (c) What is the total number of groups with different personnel, that can be accepted? [36]

This portrait in miniature of voluntary recruits might be read as a reflection of pro-military sentiment in this period. But this contemporary attention is far from universal across the books in our sample. In the same year, William Henry Harrison's *Algebra for colleges and engineering schools* references military content in the archaic context of arranging men in a regiment.

11. The men in a regiment can be arranged in a column twice as long as it is wide. If their number were 224 less they could be arranged in a hollow square 4 deep, having in each outer line of the square as many men as there were in the length of the column. Find the number of men. [16]

The strongest connection to the realities of World War II are found in a multiple-choice problem that appears in Burton Wadsworth Jones' *Elementary Concepts of Mathematics*, published in 1947:

⁴Not dissimilar to a problem about an army messenger traveling by land and by water in William Le Roy Hart's *Introduction to college algebra* from 1926 ([17])

Some Nazis are cruel. a. He is a Nazi. Hence he is cruel. b. He is cruel. Hence he is a Nazi. c. He is not a Nazi. Hence he is not cruel. d. He is not cruel. Hence he is not a Nazi. . . . [22]

Another question from this section focuses on Polynesians. Though the problem read today is disquietingly racist, the subject choice may be a reflection of recent American involvement in the islands of the South Pacific.

1. All Polynesians are brown. a. If a man is Polynesian, he is brown. b. If a man is not brown, he is Polynesian. c. In order to be brown, a man must be Polynesian. d. Whenever a man is Polynesian, he is sure to be brown. e. If a man is not brown, he cannot be Polynesian. f. If a man is not Polynesian, he is not brown.

Jones' attention to other aspects of historical and contemporary events are captured in a problem about an investment from 1922 that will mature in 1932.

On January 1, 1922, a streetcar company issued \$10,000 in 5% bonds to mature January 1, 1932. (That is, the bonds pay 5% of the face value each year and the total face value at the time of maturity.) If the bonds are to be redeemed by sinking funds at 4% interest, how much must be set aside from the company's earnings at the end of each year to provide for the interest and the retirement of the debt?

As the Great Depression begins within this interval, the reader from 1947 might have nodded in appreciation of the financial prudence of the streetcar company's risk management. In these features, *Elementary Concepts of Mathematics* is a clear example of the "real data" style of word problems, mentioned in our introduction.

While the other textbooks under consideration in this period are less detailed in their facts, in questions about transportation, airplanes are traveling slightly faster than in the previous time period, such as in William Henry Harrison Cowles and James E. Thompson's 1947 *Algebra for Colleges and Engineering Schools Second Edition*:

An airmail route is established between two cities, A and B, 600 miles apart and stops are made at an intermediate city C. One airplane leaving A for B reaches C in $2\frac{1}{2}$ hours, and another on its way from B to A reaches C in 2 hours and 48 minutes, traveling at a speed 25 miles per hour more than the first. What are the speeds of both planes? [11]⁵

Solving the problem reveals that the first plane traveled at 100 mph and the other at 125. Similarly, a plane flying “with the wind” has a rate of 150 mph in Earle Brenneman Miller’s *College Algebra* from 1950. There are dozens of questions about plane travel in *Algebra: its Big Ideas and Basic Skills*, published in 1960. The cover of the text features a photograph of eight teenagers looking upward toward a drawing of a plane moving toward the right of the page. The planes fly at various speeds, from 120 miles per hour (leaving Hickam Field, Hawaii) to jets traveling at 504 miles per hour.

A jet plane flew from Houston, Texas, to Cleveland, Ohio — a distance of 1505 miles. The plane spent 20 minutes on the ground in Cleveland. It then took off and flew 460 miles to New York. The plane’s average speed while in the air was 504 miles per hour. Find the number of hours that elapsed between the take-off in Houston and the landing in New York.

Perhaps the most striking feature of this problem to the contemporary reader is the brief layover time between flights.

Further, the period from 1941 to 1960 captures a time when personal automobile ownership was on the rise, thus car mileage and the cost of gasoline became questions of genuine interest to the average American. *Algebra for Colleges and Engineering Schools Second Edition* captures an experience of price difference across state lines.

A motorist bought 10 gallons of gasoline and 6 quarts of oil for \$3.60 and then drove to an adjoining state where gasoline was selling for $\frac{1}{2}$ c a gallon more than at home. There he bought 8 gallons of gasoline and 2 quarts of oil for \$2.16. What were the prices of gasoline and of oil in his home state? [11]

⁵The first edition is from 1935 and was not available for consultation.

While these prices might seem low at first glance, we would also need to know rates of inflation to have a meaningful comparison to gasoline costs at other times. A more straightforward comparison can be made when considering the miles per gallon question from Nowlan’s *College Algebra* in which “a car consumes 10 1/2 gallons of gasoline in traveling 196 miles” [36].

For the texts from the 1940s, women in word problems remain engaged in domestic tasks, like buying cloth with their daughters in [22] or organizing a dinner party in [35].

From a list of 12 friends, a lady selects 5 for a dinner party. What is the probability that two particular persons will be selected?⁶

In Lovincy J. Adams’ *First Course in Algebra for Colleges* from 1955, we find a Mrs. Cox who invests \$5000 and a Mrs. Moskal who “bought a lot and built a home on it” [1]. However, the other named and unnamed characters are male. Similarly, in [2], there is an unevenness between the activities of female and male characters that reflect the time of publication in 1960. Mrs. Williams buys a davenport, Catherine “can type 4 pages of copy in 38 minutes,” and Mary makes some candy. At the same time, since all unnamed professionals—e.g. salespeople, dealers, vendors—are assigned male pronouns, men engage in activities like mowing lawns, selling tennis balls, buying cows, selling books, buying bonds, dealing cars, making metal boxes, and painting houses.

These two later textbooks also include an interesting type of experimental mathematical word problem, in which the reader is instructed to go out and measure something or observe some phenomena.

Hold a candle to the left of a lens, and move a sheet of paper to the right of the lens until a distinct image of the candle appears. Measure the object distance and image distance and use $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ to calculate the focal length of the lens. [1]

Draw a street map for the neighborhood around your school. Choose two intersecting streets near your school as the reference

⁶Though beyond the bounds of this study, it is also interesting to note that the American Mortality Table presented in [35] has no reference to sex.

streets. Label that intersection $(0,0)$. Consider north and east as positive directions, and south and west as negative directions. Locate some places on your map and label the by the same system you used in Problems 12-21. [2]

These word problems directly engage students in applications of mathematical concepts. Students can also participate in *Algebra: its Big Ideas and Basic Skills* by writing their own puzzle that can be solved by an equation or equations for the other members of your class to solve” [2]. This problem is followed with a suggestion, presumably for the instructor.

SUGGESTION: Three members of the class may act as a committee to study all the puzzles. Then they can select the ten best ones and have them mimeographed for the class to solve as a later assignment.

Perhaps some of these best “puzzles” might end up in future algebra textbooks.

1961–1980

Among our textbook sample for this period are two translations of Russian algebra books from 1978 and 1979. These books—*Problem Book Algebra and Elementary Functions* and *Algebra Can Be Fun*—were both published by Mir Publishers in Moscow and purchased by the Pomona College Library. The presence of these books in our sample surprised us, especially since they are the only examples of translated texts, they were both purchased during the Cold War, and, as this research project shows, the algebraic contents might have been taught through any number of available American algebra textbooks.⁷ For the purposes of this paper, the presence of these two books has some anecdotal, but intriguing, implications for word problem content that deserve mentioning.

⁷There may be a story behind this involving the history of Pomona College’s librarians or mathematics department.

First, these books contain Russian names, money, units of measurement, and vocabulary. Some features can be seen in a combination problem from *Algebra Can Be Fun*:

At a party, 20 people danced. Mary danced with seven partners, Olga with eight, Vera with nine, and so forth up to Nina who danced with all the partners. How many men partners were there at the party? [37]

Similarly, several problems from *Problem Book Algebra and Elementary Functions* involves growing potatoes in a kolkhoz (a kolkhoz is a collective farm in the Soviet Union, and was a term unfamiliar to us until now).

641. At a certain kolkhoz, thanks to advanced methods of planting and cultivation, 680 t of potatoes are harvested from one plot of land. Another plot of land, where these advanced methods of planting and cultivation are not applied, yields the same harvest (680 t), but the total land area is larger by 45 ha, since productivity of potatoes using ordinary methods is 9 t less a hectare than when employing advanced methods. Determine the productivity of potatoes for 1 ha for each plot of land. [24]

Algebra Can Be Fun, which was written for the “curious” reader rather than a strictly classroom setting, also includes a discussion of applications to high-speed computing machines, computer-played chess, and reaching the moon in spaceship — though none of these sections contain problems to solve.

As a final note, these texts also provide an opportunity to refute the hypothesis that problems about investments and earning interest are a reflection of capitalist societies. This is shown by a problem from [24].

One part of a sum of money totaling 2000 roubles produces an annual interest of 30 roubles, while the remaining part produces 15 roubles in interest. This second part earns 1% more than the first. What percent interest does each part of the money earn per annum?

This book also contains a version of the carriage wheel problem, cited in our introduction. This suggests that the conservatism of algebra word problems is not only across time, but also geography.

Turning to the remaining four titles in our sample, published in the United States between 1961 and 1970, the word problem content remains very similar to those from earlier periods. The airplane in Charles Francis Brumfiel's *Algebra I* (1961), is faster, but it is still flying around Chicago.

One airplane leaves New York for Chicago and travels at an average speed of 540 miles per hour. A second plane leaves New York 20 minutes later and flies along the same route at an average speed of 900 miles per hour. How far from New York will the second plane overtake the first? [7]

That same year, in *Elements of Algebra*, a short book of only 160 pages, Howard Levi strongly criticizes “puzzles and busy work” as well as material that is “otherwise debased in an effort to make it alluring or give it a spurious note of accessibility” [26]. It is therefore not surprising that this text has almost no extra-mathematical word problems, and the very few that are included are stark in details and familiar in form.

Puzzle: When rowing in the direction of the current of a river, a man can travel 6 miles in an hour, whereas when rowing against the current he only travels $3/2$ miles in an hour. How fast does he row, and how fast is the current?

By contrast, the most factually detailed word problem we found in this period concerns the precise composition of lawn fertilizer in Clifford Sloyer's *Algebra and Its Applications: A Problem Solving Approach* from 1970.

A certain company manufactures two kinds of lawn fertilizer, mix A and mix B. A bag of mix A contains 4 lb of nitrogen, 2 lb of phosphoric acid, and 1 lb of potash. A bag of mix B contains 3 lb of nitrogen, 2 lb of phosphoric acid, and 4 lb of potash. The cost of a bag of mix A is \$8; the cost of a bag of mix B is \$6. An individual has determined that his lawn requires at least 18 lb of nitrogen, 10 lb of phosphoric acid, and 8 lb of potash. Determine how many bags of each should be purchased in order to provide effective fertilization and minimize the cost. [43]

Though the precise proportions are likely fictional, the ingredient list gives the impression of agricultural know-how.

In general, when names of people appear in these texts, they do not seem to come from a diverse background. For example, Brumfiel includes over a dozen word problems each with a character named Jim or John, with at least one problem referencing both characters:

12. Jim was making up an algebra problem. He said, “Joe is now 24 years old and John is 12. In how many years will Joe be three times as old as John?” Set up Jim’s problem, solve it, and interpret your solution. [7]

Other problems in this book include the names Joe, Tom, Ann, Mary and Jane.⁸ This book also exhibits the continued tendency for most of the gendered persons to be male. One, perhaps tongue-in-cheek, story problem illustrates generic naming and conservative gender roles.

The morning after being out until 3 a.m. playing an intellectual card game with his high-brow friends, Mr. Smith handed Mrs. Smith his total winnings of \$15 and remarked, “My dear, it is a good thing that I did not quit early. I am able to give you this \$15 only because between 1 a.m. and 3 a.m. I won an average of x dollars per hand for the 30 hands that we played.” Unfortunately, later in the day Mrs. Jones remarked to Mrs. Smith that she had heard that at 1 a.m. Mr. Smith had been \$75 ahead of the game. Confronted by his wife with this evidence, Mr. Smith readily admitted its truth but stoutly affirmed that he had not lied. What do you think?

Though it may be merely a reflection of the limited sample, this period exhibits a greater diversity of textual approaches than those in the earlier periods. This could reflect the growth of “new math” as a pedagogical movement during this time period, but may also be an accident of the collection at the Claremont Colleges. Though the differences between the austere *Elements of Algebra* and the enthusiasm of *Algebra Can Be Fun* are striking, both authors profess similar goals of showing mathematics as a cultural construct valuable in itself rather than as a step toward applications outside of mathematics.

⁸Ann only shows up in conjunction with Mary or Jane.

One significant difference in the two authors' approaches is the use of story-based word problems in achieving this goal. Further research might consider the number of copies sold, editions, translations as a gauge of which titles proved the most successful (though such a methodology would not capture whether the students actually walked away from their readings with an understanding of algebra).

1981–2000

Several familiar problem types continue into this period. In Murray R. Spiegel and Robert E. Moyer's *Schaum's Outline of Theory and Problems of College Algebra 2nd edition* from 1998, there are airplanes flying out of Chicago in opposite directions, but not much faster than they flew in the 1940s: one at 180 and the other at 220 miles per hour.⁹ On the other hand, John D. Baley and Martin Holstege's *Algebra: A First Course Third Edition* (first published in 1980), includes examples of planes at 360 mph as well as a problem with a very fast jet.

A private plane flying at 150 mph left Los Angeles headed for San Francisco. One hour later a jet flying at 600 mph left traveling the same route. How long will it take the jet to overtake the private plane?

Other familiar kinds of finance problems include specific references to expanding companies—AT&T, Exxon, GE, IBM, Pfizer—and new industries, most prominently, the internet [44]. Problems about the internet in Larry J. Stephens' *Algebra for the Utterly Confused* show an enthusiasm that speaks to the novelty of commercial websites (or “Internet locations”) in 2000. Many problems include fake websites for fictional companies.

You can buy a box of floppy disks from a dealer on the Internet at Computer-supplies.com for \$8 per box. The shipping cost is \$10 per order, regardless of the number of boxes you order.

⁹The first edition—written only by Spiegel—is from 1956, which may explain these numbers. Though we were unable to obtain the first edition, the dated specificity of certain problems imply that at least some problems have been added or revised.

You can buy the same box of floppies from a local computer store for \$10.50 per box. How many boxes must you order before the cost of the purchase at Computer-supplies.com becomes less than from your local computer store? [44]

New technology also shapes the suggested mode of completing word problems. In William J. Gilbert and S. A. Vanstone's *Classical Algebra*, one problem asks the reader to "write a computer program to test whether a given number is prime" and in Spiegel and Moyer's text scientific or graphing calculators are recommended [15]. On the other end of the technological spectrum, Israel M. Gelfand and Alexander Shen include the problem to "find a record of the [sic] Bach's "Well Tempered Clavier" and enjoy it" [14].

Several of the books in this sample employ specific historical facts or practical information. In *Beginning Algebra* (1996), Margaret L. Lial, E. John Hornsby, Jr., and Charles D. Miller cite sources such as the Bureau of Economic Analysis, the National Association of State Budget Officers, and the U.S. National Endowment for the Arts as references for published data. While this publication is the seventh edition, the data is up-to-date for the early-1990s, which the authors mention in their preface and is born out in "interesting and realistic" details.

In 1991, the funding for Head Start programs increased by .50 billion dollars from the funding in 1990. In 1992, the increase was .25 billion dollars over the funding in 1991. For those three years the total funding was 5.6 billion dollars. How much was funded in each of these years? (Source: U.S. Department of Health and Human Services) [27]

Other authors use specific data without references. Spiegel and Moyer mention the rate of deforestation in El Salvador and Nigeria as well as the magnitude on the Richter scale of the 1989 Loma Prieta earthquake in San Francisco. Similarly, Baley and Holstege reference the specific restrictions of the Clean Air Act in a problem about linear equations.

The maximum permissible level of particulate matter permitted by the clean air act is 75 micrograms per cubic meter of air.

An electric generating plant can operate its new, low-pollution generator for 16 hours before the maximum permissible level of air pollutants is reached. [3]

However, in these books, as well as all the other books in our sample from this period, the majority of the word problems are vague and fictional.

Evolving gender dynamics are reflected in the problems from this period. Problems in *Algebra: A First Course* specifically mention women in the workplace.¹⁰

An appliance saleswoman sold \$3,200 worth of appliances in one week. Her earnings were \$384. What was her rate of commission?

[...]

A businesswoman invested \$2,000 and made a \$300 profit. What was her rate of profit? [3]

Similarly, in a problem about investing a “financial portfolio” in *Algebra for the Utterly Confused*, a character called Lana decides how to invest “\$30,000 in corporate and municipal bonds combined” [44]. In *Beginning Algebra*, many of the word problems pertain to twentieth-century athletics, including statistics on the women winners of the Boston marathon.

Women first ran in the Boston marathon in 1972, when Nina Kuscsik of New York won the race. In 1992, the winner was Olga Markova of Russia, whose time was .8 of an hour less than Kuscsik’s in 1972. If Markova ran $\frac{4}{3}$ as fast as Kuscsik, find each runner’s speed. [27]

A problem about a grocer in [42] inadvertently suggests authorial attention to gender diversity as the grocer switches pronouns halfway through the problem.

¹⁰Though the third edition of [3] was written by two men, the first edition from 1980 was also written with Gale M. Hughes, who appears to be a woman. The more dominant presence of women in this text as compared to others may just be a coincidence.

A grocer bought a number of cans of corn for \$14.40. Later the price increased 2 cents a can and as a result she received 24 fewer cans for the same amount of money. How many cans were in his first purchase and what was the cost per can? [42]

However, this progressive shift occurred unevenly. Contrast the above with the following logic problem from *Classical Algebra*, published in 1993.

One evening, the village squire called together all the men of the village and told them that adultery had been committed in the village. Furthermore he decreed that whenever a husband found that he had been cuckolded, he was to ceremonially plunge his wife into the village pond, in the ducking stool, at noon the following day.

All the men in the village were married and well versed in logic and mathematics, including induction. As is well known in small communities, when a wife indulges in adultery, all save her husband, know about it. [15]

On the one hand, this is a ridiculous fictional tale, but its use in a classroom might lead to extra-mathematical conversations around sexist double-standards and cuckoldry. Alongside several attempts for greater female representation, most texts from this period include first and last names from a wider range of ethnic backgrounds. One page of age problems in [3] includes the names Clare, Ben, Dawn, Michael, Harry, Ricardo, Jeremy, Susanne, Marcia, Jack, Jim, John, Sally, Louise, Maria, Don, Leona, Susan, Duong, Jose, Manuel, Maria, Tony, and Lynn.

With respect to the textbook form, books from this period are the first that we found in which word problems are segregated into separate chapters rather than being integrated with the relevant mathematical content. In Josh Rappaport's *Algebra Survival Guide*, word problems are grouped by subject-matter rather than mathematical content, including age problems and distance problems. The entire section on word problems is introduced as something disliked by the reader.

Word problems ...? Aaaagggghhh!!! Panic attack! [38]

That said, the problems themselves are of the classical variety.

Overall, we see ideas in math books and word problems from 1980 to 2000 that would not be visible at a previous point in history. This includes subjects like the internet as well as the new electronic tools available for problem solving.

2001–present

The most contemporary book in our sample is also the only title in the entire study with a sole female author: Lynette Long’s *Painless Algebra* from 2016. However, this text is fairly conservative in the extra-mathematical content. Word problems include “Keisha and Martha” earning money, the distance in miles between Sarah, Joanne, and Seth in Bethesda, and the area of a rectangular swimming pool.

The newer characteristics of how word problems are treated in Long’s text—observed in the previous time period—include adding sympathetic commentary about fear of word problems.

And of course, there are those dreaded “Word Problems,” but I’ve solved them all for you, so they’re painless. [30]

A more extreme example of this is exhibited in the word problem commentary of Michael W. Kelley’s *The Idiot’s Complete Guide to Algebra: Second Edition* from 2007, in which he describes word problems as “a necessary evil of algebra, jammed in there to show you that you can use algebra in “real life.”” [23]. However, Kelley makes no attempt to write “real life” word problems, and criticizes the uselessness of the word problems he does include.

Have you ever heard of a word problem like this one? “Train A heads north at an average speed of 95 miles per hour, leaving its station at the precise moment another train, Train B, departs a different station, heading south at an average speed of 110 miles per hour. If these trains are inadvertently placed on the same track and start exactly 1,300 miles apart, how long until they collide?”

If that problem sounds familiar, it's probably because you watch a lot of television (like me). Whenever TV shows mention math, it's usually in the context of a main character trying (but failing miserably) to solve the classic "impossible train problem." I have no idea why that is, but time and time again, this problem is singled out as the reason people hate math so much.¹¹

Bob Miller's *Bob Miller's Algebra for the Clueless: Algebra: Second Edition*, employs the same technique of critiquing word problems within the word problem:

How many pounds of peanuts, selling at 60 cents a pound must be mixed with 12 pounds of walnuts selling at 90 cents a pound to give a mixture at 70 cents a pound? Can you imagine anybody actually doing this? Oh well, let's solve it. [34]

Like Kelley, Miller's problems are familiar, including a problem about two planes leaving Chicago (one at 480 and the other at 520 mph).

There are no notable new technologies cited after 2001, but some authors include questions that concern current affairs. A question in Jerry Howett's *Algebra and Geometry* cites results of a survey on "term limits for elected officials" [19]. Robert Blitzer *Intermediate Algebra for College Students* from 2002 presents detailed tables of concrete data to encourage reading graphs and finding trends. Three examples from the beginning, middle, and end of the text capture this style of word problems [5].

The caseload of Alzheimer's disease in the US is expected to explode as baby boomers head into their later years. The graph shows the percentage of Americans with the disease, by age. Describe the trend shown in the graph.

[...]

Let x represent the number of years after 1995 and let y represent the percentage of seniors who used cocaine. Draw a line that fits the data.

¹¹Notably, though there were many problems about trains, no collision problems were found in any of the sampled texts from earlier periods.

[...]

Through the end of 1991, 200000 cases of AIDS had been reported to the Centers for Disease Control in the United States. By the end of 1998, the number had grown to 680000. The exponential growth function describes the thousands of AIDS cases in the United States t years after 1991. Use the fact that 7 years after 1991 there were 680 thousand cases to find k to three decimal places.

Blitzer also includes classic problems, such as the the rate in which a pool is filled by two pipes.

While Kelley and Miller stick with conservative word problem and Blitzer incorporates contemporary data, a hybrid style of word problem updating can be found in *Life of Fred Beginning Algebra* written by Stanley F. Schmidt. The entire text in an extended narrative about a character named Fred who teaches mathematics and embodies techniques of algebra. Word problems at the end of each chapter describe the people in Fred's life and the mathematics they encounter. One of Fred's students is named Joe, and a love-interest is Darlene. As the plot and mathematical skills progress, the reader returns to Joe and Darlene in various scenarios.

Joe's great-great-great-great-great-great grandfather was Joseph Priestly. Joe was named after his famous ancestor who had discovered the chemical element oxygen. Also in honor of "old Joseph" (as the family liked to call him) our young Joe had become a chemistry major and next semester would take calculus from Fred.

[...]

At 5:45 p.m. Joe had won a total of \$126, but ten minutes later his total was -\$17. What was his change in wealth during those ten minutes?

[...]

Darlene was reading the novel *Gone with the Wind* in which the heroine, Scarlet, had a waist size of 18". Assuming her waist was circular, approximately how thick was Scarlet at her waist?

[...]

Betty and Darlene made cookies for the Math Faire. [39]

While *Life of Fred Beginning Algebra* is unique in its commitment to a fictional narrative structure, David Alan Herzog's *Teach Yourself Visually Algebra* from 2008 employs a similar approach to word problems. The problems are specific in detail, but entirely fictional and seem to describe the collective activities of an uncannily mathematical suburbia, in which “Marge and Karen both collect crackle-glass items”—Karen has 342 and Marge has 855, Greg and Suzanne “varnish his living room floor,” and Jason, Ian, and Dylan “seal a driveway” [18]. Unlike in [39], these fictional suburban scenarios are scattered around Herzog's text and might remain unnoticed by the typical reader.

As an example of the avoidance style identified in our introduction, Lawrence Leff's *College Algebra: Second Edition* has almost no word problems except in the section on growth and decay [25].

Conclusion

As modes of transport progressed from carriages to airplanes, so did algebra word problems (although the problem of comparing front and back carriage wheel rotations does not have a simple analogue today).¹² Technological innovations like the telegraph, antifreeze, and the internet also seeped into the backgrounds of realistic mathematical scenarios, though world events appear to have had less of an impact. Some changes were based more on style than social contexts. For instance, our study suggests that proper names of fictional characters began to appear in word problems around the 1920s. Along the way, more women entered as subjects and today seem to comprise about half the people investing money and varnishing floors.

¹²There is a disassembled carriage problem in *Algebra: its Big Ideas and Basic Skills* (1960).

A wheel 6 inches in diameter makes 84 revolutions in rolling a certain distance. How many revolutions will a wheel 9 inches in diameter make in rolling the same distance? [2]

Not so for textbook writers, all but two of the fifty-six textbook authors in this study are male.¹³ In this, and other aspects, textbooks and their word problems seem to be a fairly conservative medium.

As mentioned at the beginning of this article, the value of this study was in the process of research more so than the individual results. Looking forward, a more historically rigorous corpus could be used to evaluate questions that emerged along the way. Certainly, it remains to be determined whether any of the temporal patterns observed above would be captured in a more representative sample or are merely accidental features of the peculiarities of this study. Further research could also focus on particular trends suggested here. A knowledge of inflation, the price of items, and banking history could determine the extent to which word problems reflect economic realities. Similarly, a more systematic study of the fictional names in word problems would benefit from comparative data on diversity in student populations in algebra classrooms over time. On a more superficial level, we wondered why there were so many plane problems about Chicago as compared to other cities and when algebra textbook authors began to assume that their readers hated word problems. Finally, for texts that ran multiple editions over multiple decades it would be interesting to observe to what extent word problems varied or stayed the same.

Maybe in the next twenty years, we will see an emphasis on environmental issues, political tragedies, or some other prevalent issue that we observe on a daily basis today. Surely, historians of the distant future will wonder at our apparent fascination with a traveler leaving a city at some constant speed, who will eventually be overtaken by another constant traveler at a time as yet unknown.

A. Texts Organized Chronologically by Period

Texts marked with * are available as print books through the Claremont Colleges Library.

¹³An anonymous reviewer noted that Mary Dolciani wrote a series of popular algebra textbooks, which were published in numerous editions from the 1970s to 1990s, but these texts were not available through our sources.

1901 – 1920

G. A. Wentworth and G. A. Hill, *Wentworth & Hill's exercise manuals, 2. Algebra*, Boston, Ginn & Company, 1901 (first edition 1884).

George Chrystal, *Algebra: An Elementary Textbook*, London, Charles Black, 1904 (first edition 1886).*

Charles Smith and Irving Stringham, *Elementary Algebra for the Use of Schools and Colleges*, New York, The Macmillan Company, 1904 (first edition 1895).

H. E. Slaught and N. J. Lennes, *First Principles of Algebra: Complete Course*, Boston, Allyn and Bacon, 1912.

Stimson J. Brown, *Practical Algebra (revised)*, Annapolis, The United States Naval Institute, 1913 (first edition 1908).

Franklin Turner Jones, *Problems and Questions on Algebra*, Cleveland, University Publishing Company, 1913.

1921–1940

W. B. Ford, *A Brief Course in College Algebra*, New York, The Macmillan Company, 1922.

P. Ligda, *The Teaching of Elementary Algebra*, Boston, Houghton Mifflin, 1925.

George Chrystal, *Algebra: Part I, Fifth Edition*, London, A. & C. Black, 1926 (first edition 1886).

F. Engelhardt and L. Haertter, *First Course in Algebra*, Philadelphia, The John C. Winston Company, 1926.

W. L. Hart, *College Algebra Minnesota*, D. C. Heath and Company, 1926.*

Nelson A. Jackson, *Beginning Algebra*, Massachusetts, Mount Hermon School, 1933.

G. Bartoo and J. Osborn, *First Year Algebra*, Missouri, Webster Publishing, 1937.

1941–1960

Frank Millett Morgan, *College Algebra*, New York, American Book Company, 1943.

Frederick Stanley Nowlan, *College Algebra*, New York, McGraw-Hill Book Company, 1947.

William Henry Harrison Cowles and James E. Thompson, *Algebra for Colleges and Engineering Schools Second Edition*, New York, D. Van Nostrand, 1947 (first edition 1935).

Burton Wadsworth Jones, *Elementary Concepts of Mathematics*, New York, Macmillan Company, 1947.*

Earle Brenneman Miller, *College Algebra*, New York, Ronald Press Company, 1950.

Lovincy J. Adams, *First Course in Algebra for Colleges*, New York, Holt, 1955.*

Daymond J. Aiken, Kenneth B. Henderson, and Robert E. Pingry, *Algebra: Its Big Ideas and Basic Skills*, New York, McGraw-Hill, 1960.*

1961–1980

Charles Francis Brumfiel, *Algebra I*, Reading, MA, Addison-Wesley Pub. Co., 1961.

Howard Levi, *Elements of Algebra*, New York, Chelsea Publishing Company, 1961.*

Irwin Miller and Simon Green, *Algebra and Trigonometry*, New Jersey, Prentice-Hall, 1962.

Clifford W. Sloyer, *Algebra and Its Applications: A Problem Solving Approach* Reading, M.A., Addison-Wesley Publishers, 1970.*

A. Kutepov and A. Rubanov, *Problem Book Algebra and Elementary Functions*, Moscow, Mir Publishers, 1978.*

Yakov Isidorovich Perelman *Algebra Can Be Fun* trans. George Yankovsky, Moscow, Mir Publishers, 1979.*

1981–2000

John D. Baley and Martin Holstege, *Algebra: A First Course Third Edition*, Belmont, Wadsworth Publishing Company, 1990 (first edition 1980).

Israel M. Gelfand and Alexander Shen, *Algebra*, Boston, Birkhäuser, 1993.*

William J. Gilbert and S. A. Vanstone, *Classical Algebra* Waterloo, University of Waterloo, Centre for Education in Mathematics and Computing, 1993.*

Margaret L. Lial, E. John Hornsby, Jr., Charles D. Miller, *Beginning Algebra*, New York, Harper Collins, 1996 (first edition 1969).*

Murray R. Spiegel and Robert E. Moyer, *Schaum's Outline of Theory and Problems of College Algebra 2nd edition* New York, McGraw-Hill, 1998 (first edition 1956).

Josh Rappaport, *Algebra Survival Guide* Santa Fe, Singing Turtle Press, 2000.

Larry J. Stephens, *Algebra for the Utterly Confused*, New York, McGraw-Hill, 2000.

2001–present

Robert Blitzer, *Intermediate Algebra for College Students*, Prentice-Hall, New Jersey, 2002.

Stanley F. Schmidt, *Life of Fred Beginning Algebra* Reno, Polka Dot Publishing, 2002.*

Jerry Howett, *Algebra and Geometry*, Syracuse, NY, New Readers, 2003.

Lawrence S. Leff, *College Algebra: Second Edition*, Barron's Educational Series, New York, 2005 (first edition 1995).

Michael W. Kelley, *The Idiot's Complete Guide to Algebra: Second Edition*, Penguin Group, New York, 2007 (first edition 2004).*

Bob Miller, *Bob Miller's Algebra for the Clueless: Algebra: Second Edition*, McGraw Hill, New York, 2007.*

David Alan Herzog, *Teach Yourself Visually Algebra*, Wiley Publishing, Inc., New Jersey, 2008.

Lynette Long, *Painless Algebra: Fourth Edition*, Barron's Educational Series, New York, 2016 (first edition 1998).

References

- [1] Lovincy J. Adams, *First Course in Algebra for Colleges*, New York, Holt, 1955.
- [2] Daymond J. Aiken, Kenneth B. Henderson, and Robert E. Pingry, *Algebra: Its Big Ideas and Basic Skills*, New York, McGraw-Hill, 1960.
- [3] John D. Baley and Martin Holstege, *Algebra: A First Course Third Edition*, Belmont, Wadsworth Publishing Company, 1990.
- [4] G. Bartoo and J. Osborn, *First Year Algebra*, Missouri, Webster Publishing, 1937.
- [5] Robert Blitzer, *Intermediate Algebra for College Students*, Prentice-Hall, New Jersey, 2002.
- [6] Stimson J. Brown, *Practical Algebra (revised)*, Annapolis, The United States Navel Institute, 1913.
- [7] Charles Francis Brumfiel, *Algebra I*, Reading, MA, Addison-Wesley Pub. Co., 1961.
- [8] Clark, Kathleen Michelle. "History of mathematics: illuminating understanding of school mathematics concepts for prospective mathematics teachers" *Educational Studies in Mathematics* **81**, 2012, pp. 67–84.
- [9] George Chrystal, *Algebra: An Elementary Textbook*, London, Charles Black, 1904.
- [10] George Chrystal, *Algebra: Part I*, London, A. & C. Black, 1926.
- [11] William Henry Harrison Cowles and James E. Thompson, *Algebra for Colleges and Engineering Schools Second Edition*, New York, D. Van Nostrand, 1947.
- [12] F. Engelhardt and L. Haertter, *First Course in Algebra*, Philadelphia, The John C. Winston Company, 1926.

- [13] W. B. Ford, *A Brief Course in College Algebra*, New York, The Macmillan Company, 1922.
- [14] Israel M. Gelfand and Alexander Shen, *Algebra*, Boston, Birkhäuser, 1993.
- [15] William J. Gilbert and S. A. Vanstone, *Classical Algebra* Waterloo, University of Waterloo, Centre for Education in Mathematics and Computing, 2000.
- [16] William Henry Harrison Cowles and James E. Thompson, *Algebra for Colleges and Engineering Schools Second Edition*, New York, D. Van Nostrand, 1947.
- [17] W. L. Hart, *College Algebra* Minnesota, D. C. Heath and Company, 1926.
- [18] David Alan Herzog, *Teach Yourself Visually Algebra*, Wiley Publishing, Inc., New Jersey, 2008.
- [19] Jerry Howett, *Math Sense and Geometry*, Syracuse, NY, New Readers, 2003.
- [20] Nelson A. Jackson, *Beginning Algebra*, Massachusetts, Mount Hermon School, 1933.
- [21] Franklin Turner Jones, *Problems and Questions on Algebra*, Cleveland, University Publishing Company, 1913.
- [22] Burton Wadsworth Jones, *Elementary Concepts of Mathematics*, New York, Macmillan Company, 1947.
- [23] Michael W. Kelley, *The Idiot's Complete Guide to Algebra: Second Edition*, Penguin Group, New York, 2007.
- [24] A. Kutepov and A. Rubanov, *Problem Book Algebra and Elementary Functions*, Moscow, Mir Publishers, 1978.
- [25] Lawrence S. Leff, *College Algebra: Second Edition*, Barron's Educational Series, New York, 2005.

- [26] Howard Levi, *Elements of Algebra*, New York, Chelsea Publishing Company, 1961.
- [27] Margaret L. Lial, E. John Hornsby, Jr., Charles D. Miller, *Beginning Algebra*, New York, Harper Collins, 1996.
- [28] P. Ligda, *The Teaching of Elementary Algebra*, Boston, Houghton Mifflin, 1925.
- [29] Lim, Siew Yee and Elaine Chapman, “Effects of using history as a tool to teach mathematics on students’ attitudes, anxiety, motivation and achievement in grade 11 classrooms” *Educational Studies in Mathematics* **90**, 2015, pp. 189–212.
- [30] Lynette Long, *Painless Algebra: Fourth Edition*, Barron’s Educational Series, New York, 2016.
- [31] McBride, Cecil and James H. Rollins, “The Effects of History of Mathematics on Attitudes towards Mathematics of College Algebra Students” *Journal for Research in Mathematics Education* **8**(1), January 1977, pp. 47–61.
- [32] Bob Miller, *Bob Miller’s Algebra for the Clueless: Algebra: Second Edition*, McGraw Hill, New York, 2007.
- [33] Earle Brenneman Miller, *College Algebra*, New York, Ronald Press Company, 1950.
- [34] Irwin Miller and Simon Green, *Algebra and Trigonometry*, New Jersey, Prentice-Hall, 1962.
- [35] Frank Millett Morgan, *College Algebra*, New York, American Book Company, 1943.
- [36] Frederick Stanley Nolan, *College Algebra*, New York, McGraw-Hill Book Company, 1947.
- [37] Yakov Isidorovich Perelman *Algebra Can Be Fun* trans. George Yankovsky, Moscow, Mir Publishers, 1979.
- [38] Josh Rappaport, *Algebra Survival Guide* Santa Fe, Singing Turtle Press, 2000.

- [39] Stanley F. Schmidt, *Life of Fred Beginning Algebra* Reno, Polka Dot Publishing, 2002.
- [40] H. E. Slaught and N. J. Lennes, *First Principles of Algebra: Complete Course*, Boston, Allyn and Bacon, 1912.
- [41] Charles Smith and Irving Stringham, *Elementary Algebra for the Use of Schools and Colleges*, New York, The Macmillan Company, 1904.
- [42] Murray R. Spiegel and Robert E. Moyer, *Schaum's Outline of Theory and Problems of College Algebra 2nd edition* New York, McGraw-Hill, 1998.
- [43] Clifford W. Sloyer, *Algebra and Its Applications: A Problem Solving Approach* Reading, M.A., Addison-Wesley Publishers, 1970.
- [44] Larry J. Stephens, *Algebra for the Utterly Confused*, New York, McGraw-Hill, 2000.
- [45] G. A. Wentworth and G. A. Hill, *Wentworth & Hill's exercise manuals, 2. Algebra*, Boston, Ginn & Company, 1901.
- [46] Michael Willers, *Algebra, the x and y of Everyday Math*, New York, Metro Books, 2009.